Incremental Slow Feature Analysis with Indefinite Kernel for Online Temporal Video Segmentation

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1. Introduction

Slow Feature Analysis is a subspace learning method which

- has been inspired by the human visual system [1],
- is still seldom found in computer vision, and
- has been applied to unsupervised activity analysis [2].

Our contributions are

- the proposal of kernel SFA in Krein space,
- the formulation of an accurate incremental kernel SFA, and
- SFA's first online temporal video segmentation algorithm, which does not require prior training nor prior data specific knowledge.

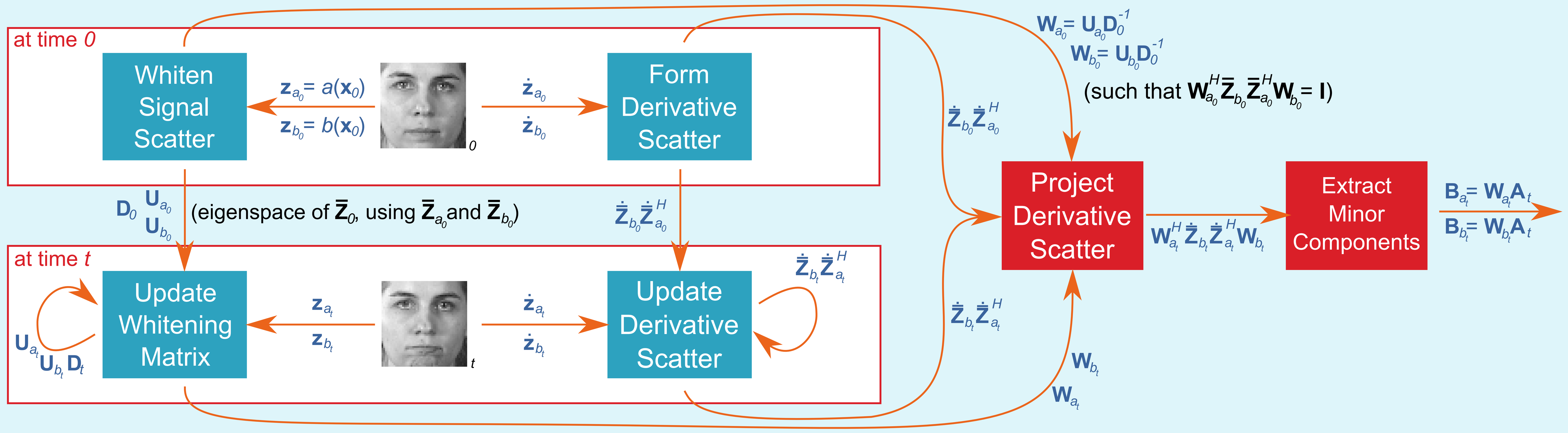

Figure 2: The incremental SFA with our indefinite Kernel.

2. Kernel SFA in Krein Space

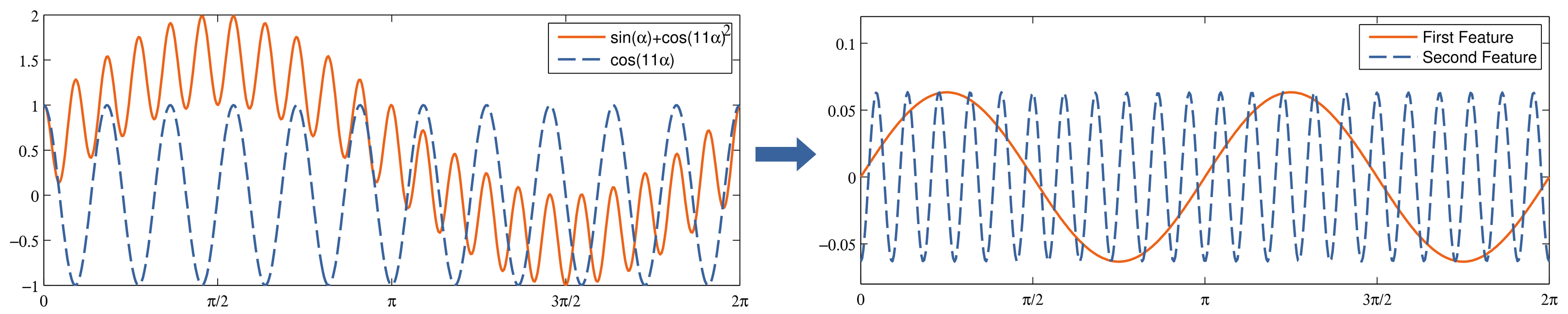

Figure 1: SFA extracts the slow features from the input signal.

Using the centered signal $\bar{\mathbf{Z}}$ and its derivative $\dot{\mathbf{Z}}$, SFA is solved by finding the projection $\mathbf{B}$, which minimizes the signal's change:

$$\min_{\mathbf{B}} tr\left(\left(\mathbf{B}^H\bar{\mathbf{Z}}\bar{\mathbf{Z}}^H\mathbf{B}\right)^{-1}\mathbf{B}^H\dot{\mathbf{Z}}\dot{\mathbf{Z}}^H\mathbf{B}\right)$$

A Krein space, $\mathcal{K}$

- provides a geometry for classifiers with non-positive kernels [3],
- is described by an inner product $\langle .,. \rangle_{\mathcal{K}} : \mathcal{K} \times \mathcal{K} \rightarrow \mathbb{C}$, for which

$$\forall \mathbf{x}, \mathbf{y} \in \mathcal{K} \quad \langle \mathbf{x}, \mathbf{y} \rangle_{\mathcal{K}} = \overline{\langle \mathbf{y}, \mathbf{x} \rangle}_{\mathcal{K}}$$

$$\forall \mathbf{x}, \mathbf{y}, \mathbf{z} \in \mathcal{K}, c_1, c_2 \in \mathbb{R} \quad \langle c_1\mathbf{x} + c_2\mathbf{z}, \mathbf{y} \rangle_{\mathcal{K}} = c_1\langle \mathbf{x}, \mathbf{y} \rangle_{\mathcal{K}} + c_2\langle \mathbf{z}, \mathbf{y} \rangle_{\mathcal{K}}$$

- consists of a positive and a negative Hilbert space, $\mathcal{K}_+$ and $\mathcal{K}_-$

$$\forall \mathbf{x}, \mathbf{y} \in \mathcal{K} \qquad \langle \mathbf{x}, \mathbf{y} \rangle_{\mathcal{K}} = \langle \mathbf{x}_+, \mathbf{y}_+ \rangle_{\mathcal{K}_+} - \langle \mathbf{x}_-, \mathbf{y}_- \rangle_{\mathcal{K}_-}$$

- has an associated Hilbert space, $|\mathcal{K}|$, which can be expressed using its fundamental symmetry $\mathbf{J} = \mathbf{J}^{-}1 = \mathbf{J}^T$

$$\mathbf{x}^*\mathbf{y} \triangleq \langle \mathbf{x}, \mathbf{y} \rangle_{\mathcal{K}} = \mathbf{x}^H\mathbf{J}\mathbf{y} = \langle \mathbf{J}\mathbf{x}, \mathbf{y} \rangle_{|\mathcal{K}|}$$

We formulate non-positive KSFA's projection as $\mathbf{B} = \bar{\mathbf{Z}}\tilde{\mathbf{B}}$, and get

$$\min_{\tilde{\mathbf{B}}} tr\left(\left(\tilde{\mathbf{B}}^H\bar{\mathbf{Z}}^*\bar{\mathbf{Z}}\bar{\mathbf{Z}}^*\bar{\mathbf{Z}}\tilde{\mathbf{B}}\right)^{-1}\tilde{\mathbf{B}}^H\bar{\mathbf{Z}}^*\dot{\mathbf{Z}}\dot{\mathbf{Z}}^*\bar{\mathbf{Z}}\tilde{\mathbf{B}}\right)$$

We simplify the problem by finding $\tilde{\mathbf{W}}$, such that $\tilde{\mathbf{W}}^H\bar{\mathbf{Z}}^*\bar{\mathbf{Z}}\bar{\mathbf{Z}}^*\bar{\mathbf{Z}}\tilde{\mathbf{W}} = \mathbf{I}$:

$$\min_{\mathbf{A}} tr\left(\mathbf{A}^H\tilde{\mathbf{W}}^H\bar{\mathbf{Z}}^*\dot{\mathbf{Z}}\dot{\mathbf{Z}}^*\bar{\mathbf{Z}}\tilde{\mathbf{W}}\mathbf{A}\right) \qquad \text{subject to } \mathbf{A}^H\mathbf{A} = \mathbf{I}$$

SFA's projection is then given by $\mathbf{B} = \bar{\mathbf{Z}}\tilde{\mathbf{B}} = \bar{\mathbf{Z}}\tilde{\mathbf{W}}\mathbf{A}$.

3. Robust Gradient-based Kernel

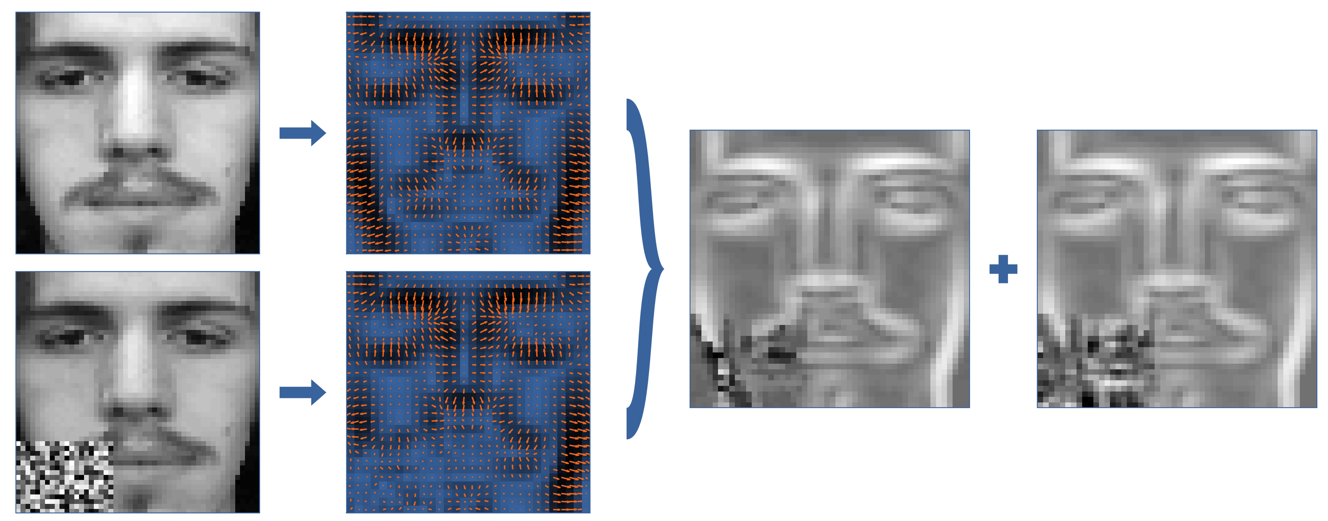

Figure 3: Visualization of our kernel when corruption is present.

Our robust gradient-based kernel [4] , $k(\mathbf{x}_p, \mathbf{x}_q)$, is given by

$$\frac{\sum_{c=1}^{d} \mathbf{R}_p(c)\left(\cos(\Delta\theta(c)) - i\sin(\Delta\theta(c))\right)}{2\sqrt{\sum_{c=1}^{d}\mathbf{R}_p^2(c)d}} + \frac{\sum_{c=1}^{d} \mathbf{R}_q(c)\left(\cos(\Delta\theta(c)) - i\sin(\Delta\theta(c))\right)}{2\sqrt{\sum_{c=1}^{d}\mathbf{R}_q^2(c)d}}$$

with gradient magnitude $\mathbf{R}$, angle difference $\Delta\theta$, dimensionality d. We exploit the kernel's special form, $k(\mathbf{x}_p, \mathbf{x}_q) = a(\mathbf{x}_p)^H b(\mathbf{x}_q)$.

4. Temporal Video Segmentation

Frames with significant changes reveal the segmentation.

Change **is the difference in slow feature space**

$$c_t(\mathbf{z}_i) = \mathbf{B}_t^H(\mathbf{z}_i - \mathbf{z}_{i-1})(\mathbf{z}_i - \mathbf{z}_{i-1})^H\mathbf{B}_t$$

Significance **is the ratio of the current and the mean change**

$$\frac{(t-1)c_{t-1}(\mathbf{z}_t)}{\sum_{i=1}^{t-1} c_{k_{t-1}}(\mathbf{z}_i)}$$

Note, $\sum_{i=1}^{t-1} c_{k_{t-1}}(\mathbf{z}_i)$ is the sum of eigenvalues in $\mathbf{W}^H\dot{\mathbf{Z}}\dot{\mathbf{Z}}^H\mathbf{W}$, which is part of online SFA. Thus, we do not require previous samples.

5. Results

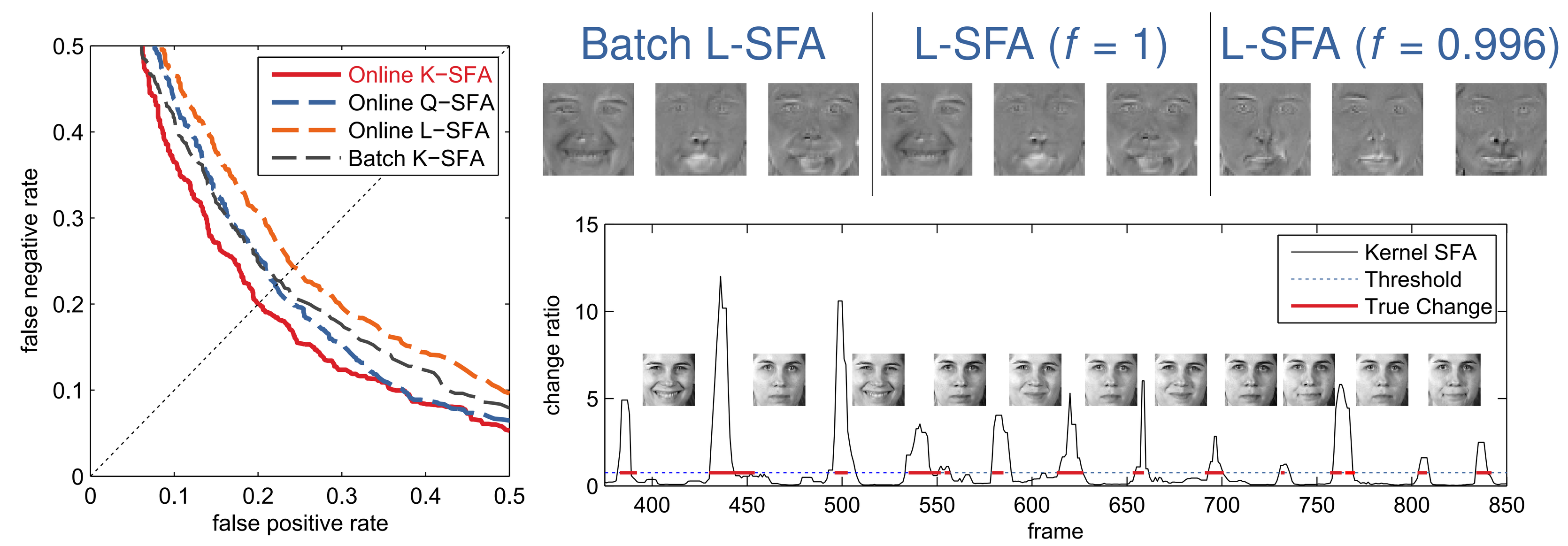

Figure 4: ROC, SFA's projection after 900 frames and change detection results.

Our approach (Online K-SFA)

- correctly identifies the changes in several videos for most cases,
- performs best and computes fasted for the Krein space kernel,
- successfully calculates the exact SFA incrementally.

6. Further Information

Visit `http://www.doc.ic.ac.uk/~sl609/sfa/`

- to access our evaluation with video results and
- to download the source code of our Incremental SFA.

S. Liwicki, S. Zafeiriou and M. Pantic, "Incremental Slow Feature Analysis with Indefinite Kernel for Online Temporal Video Segmentation," ACCV'12.

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[2] A. Nater, H. Grabner, and L. Van Gool, "Temporal Relations in Videos for Unsupervised Activity Analysis," in *Mach. Learning*, 2004, pp. 78–86.
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[4] S. Liwicki, S. Zafeiriou, G. Tzimiropoulos, and M. Pantic, "Efficient Online Subspace Learning with an Indefinite Kernel for Visual Tracking and Recognition," *IEEE Trans. Neu. Net. Learn. Systems*, vol. 23, no. 10, pp. 1624–1636, Oct 2012.